

Malitsky

MATH 114 – FINAL EXAM

May 15, 2012

Your name:

Circle your TA's name:

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- **Be sure to show your work and explain what you did. You will receive reduced or zero credit for unsubstantiated answers.**
- **No books or calculators. You may refer to notes you have brought on one sheet of paper, as announced in class.**
- **Circle your answer.**

| problem | possible score | score |
|---------|----------------|-------|
| 1 | 5, 5 | |
| 2 | 10 | |
| 3 | 5, 5, 5 | |
| 4 | 5, 10 | |
| 5 | 10 | |
| 6 | 15 | |
| 7 | 10, 10 | |
| 8 | 15 | |
| 9 | 15 | |
| 10 | 15 | |
| Total | 140 | |

1. Suppose a circle has circumference 12π .

a) Find its radius.

$$C = \pi \cdot d$$

$$12\pi = \pi \cdot d$$

$$d = 12$$

$$r = \frac{d}{2}$$

$$r = \frac{12}{2}$$

$$r = 6$$

b) Find its area.

$$A = \pi \cdot r^2$$

$$A = \pi \cdot 6^2$$

$$A = 36\pi$$

2. For the function $F(x)$ below determine whether it is even, odd, or neither. Explain your answer.

$$F(x) = x + \cos x$$

$$F(-x) = -x + \cos(-x) = -x + \cos(x)$$

$$F(-x) \neq -F(x)$$

$$F(-x) \neq F(x)$$

It is neither.

3. Let $f(x) = \begin{cases} \frac{x}{|x|}, & \text{if } x \neq 0 \\ 0, & \text{if } x = 0 \end{cases}$

a) Compute $f(5)$ and $f(-5)$.

$$f(5) = \frac{5}{|5|} = \frac{5}{5} = 1$$

$$f(-5) = \frac{-5}{|-5|} = \frac{-5}{5} = -1$$

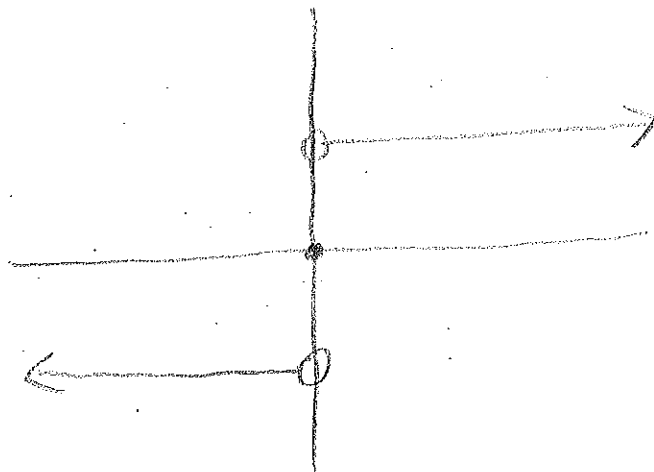
$$\begin{array}{l} f(5) = 1 \\ f(-5) = -1 \end{array}$$

b) Find the domain and the range of the function $f(x)$.

$$D: (-\infty, \infty)$$

$$R: f(x) = -1, 0, 1$$

c) Graph the function.



4. Let $f(x) = -3x + 5$.

a) Compute $f(\frac{5}{2})$ and $f(\frac{5}{2} + h)$.

$$\begin{aligned} f\left(\frac{5}{2}\right) &= -3\left(\frac{5}{2}\right) + 5 \\ &= -\frac{15}{2} + \frac{10}{2} \end{aligned}$$

$$\boxed{f\left(\frac{5}{2}\right) = -\frac{5}{2}}$$

$$\begin{aligned} f\left(\frac{5}{2} + h\right) &= -3\left(\frac{5}{2} + h\right) + 5 \\ &= -\frac{15}{2} - 3h + \frac{10}{2} \\ &= -\frac{5}{2} - 3h \end{aligned}$$

$$\boxed{f\left(\frac{5}{2} + h\right) = -\left(\frac{5 + 6h}{2}\right)}$$

b) Compute $\frac{f(\frac{5}{2} + h) - f(\frac{5}{2})}{h}$. Simplify your answer as much as possible.

$$\frac{f\left(\frac{5}{2} + h\right) - f\left(\frac{5}{2}\right)}{h} = \frac{-\frac{5}{2} - 3h + \frac{5}{2}}{h} = -3$$

$$\boxed{\frac{f\left(\frac{5}{2} + h\right) - f\left(\frac{5}{2}\right)}{h} = -3}$$

5. Find all numbers x that satisfy the given equation

$$e^{-2\ln x} = 5$$

$$\ln e^{-2\ln x} = \ln 5$$

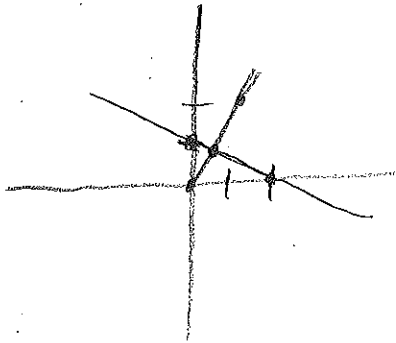
$$-2\ln x = \ln 5$$

$$\ln x = \ln 5^{-1/2}$$

$$x = \frac{1}{\sqrt{5}}$$

$$x = \frac{\sqrt{5}}{5}$$

6. Find the distance from the origin to the line $y = -\frac{1}{2}x + 1$.



Find a line \perp to
 $y = -\frac{1}{2}x + 1$ that passes
through the origin.

$$m = -\frac{1}{(-\frac{1}{2})} = 2$$

$$0 = 2 \cdot (0) + b$$

$$b = 0$$

$$y = 2x$$

Take distance from origin to intersection

$$2x = -\frac{1}{2}x + 1$$

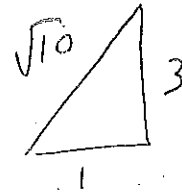
$$\frac{5}{2}x = 1$$

$$x = \frac{2}{5}, \quad y = \frac{4}{5}$$

$$d = \sqrt{\left(\frac{2}{5}\right)^2 + \left(\frac{4}{5}\right)^2} = \sqrt{\frac{20}{25}} = \frac{2\sqrt{5}}{5}$$

$$d = \frac{2\sqrt{5}}{5}$$

7. Suppose a triangle has sides $a = 1$, $b = 3$, and $c = \sqrt{10}$.



a) Find its area.

$$A = \frac{1}{2} a \cdot b \cdot \sin C$$

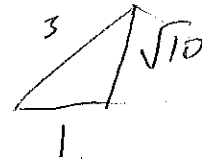
$$c^2 = a^2 + b^2 - 2ab \cdot \cos C$$

$$10 = 1 + 9 - 2(1)(3)\cos C$$

$$0 = \cos(C)$$

$$\arccos(0) = C$$

$$C = \frac{\pi}{2}$$



$$A = \frac{1}{2} \cdot 1 \cdot 3 \cdot \sin \frac{\pi}{2}$$

$$A = \frac{3}{2}$$

b) Find the angle C opposite the side c . Write your answer in degrees and in radians.

$$C = \frac{\pi}{2}, 90^\circ$$

8. Find the exact value of $\cos(2x)$ and $\tan(2x)$, if $\sin x = \frac{4}{5}$, $\pi/2 < x < \pi$.

$$\cos(2x) = 1 - 2\sin^2 x$$

$$= 1 - 2 \cdot \left(\frac{4}{5}\right)^2$$

$$= 1 - 2 \cdot \frac{16}{25}$$

$$= 1 - \frac{32}{25}$$

$$= \frac{25 - 32}{25}$$

$$\boxed{\cos(2x) = \frac{-7}{25}}$$

$$\tan(2x) = \frac{2 \cdot \tan(x)}{1 - \tan^2 x}$$

$$\tan(2x) = \frac{\sin(2x)}{\cos(2x)}$$

$$= \frac{4/5}{-7/25}$$

$$= -\frac{4}{5} \cdot \frac{25}{7}$$

$$\tan(2x) = \frac{-100}{35} = \frac{-20}{7}$$

$$\boxed{\tan(2x) = \frac{-20}{7}}$$

9. Find the exact value of $\cos\left(\frac{7\pi}{12}\right)$.

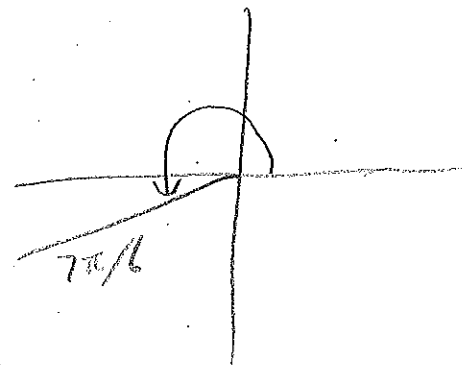
$$\cos\left(\frac{7\pi}{12}\right) = \cos\left(\frac{1}{2} \cdot \frac{7\pi}{6}\right)$$

$$= -\sqrt{\frac{1 + \cos\frac{7\pi}{6}}{2}}$$

$$= -\sqrt{\frac{1 + -\sqrt{3}/2}{2}}$$

$$= -\sqrt{\frac{2 - \sqrt{3}}{4}}$$

$$\boxed{\cos\left(\frac{7\pi}{12}\right) = -\frac{\sqrt{2 - \sqrt{3}}}{2}}$$



10. A movie theatre sells 250 tickets to a show. The adult price is \$8, and the student price is \$6, and the box office took in \$1,920. How many of each type of ticket was sold? (One way of doing this problem is writing and solving a system of two linear equations in two variables).

$$x + y = 250$$

$$y = 250 - x$$

$$8x + 6y = 1920$$

$$8x + 6(250 - x) = 1920$$

$$2x = 420$$

$$\boxed{\begin{array}{l} x = 210 \\ y = 40 \end{array}}$$