PROBLEM SET 1B

1. $15^{2x+1} = 17^{x+1}$

$\log 15^{2x+1} = \log 17^{x+1}$

$(2x+1)\log 15 = (x+1)\log 17$

$2x\log 15 + \log 15 = x\log 17 + \log 17$

$2x\log 15 - x\log 17 = \log 17 - \log 15$

$x(2\log 15 - \log 17) = \log 17 - \log 15$

$x = \dfrac{\log 17 - \log 15}{2\log 15 - \log 17}$

2. Find all values of x in the interval $[0, 2\pi]$ which satisfy the inequality. Write your solution as a union of intervals.

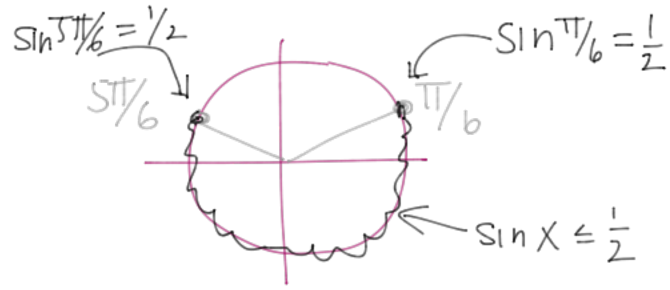
$$\sin x \leq 0.5$$

$\sin x = 0.5$ has solutions $x = \pi/6, 5\pi/6$

check & shade

gives $[0, \pi/6] \cup [5\pi/6, 2\pi]$

OR: Just look at unit circle



3. Find the domain of the function. Express your answer in interval notation.

$$f(x) = \ln\left(x - \frac{x^2 - 21}{x + 7}\right)$$

ln is only defined for positive numbers, so we need $x - \dfrac{x^2-21}{x+7} > 0$

To solve inequalities like this, we find where the sign changes, which happens when the function is 0 or undefined. It is undefined at $x = -7$. Now solve:

$x - \dfrac{x^2-21}{x+7} = 0$

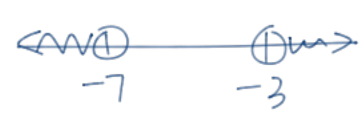
$x = \dfrac{x^2-21}{x+7}$

$x^2 + 7x = x^2 - 21$

$7x = -21$

$x = -3$

Now test on number line:



4. Sketch the graph (include 2 full periods) of the function. Find the exact value of all x-intercepts.

$$y = 2\cos(\pi x - \frac{\pi}{4})$$

Find the x-intercepts:

$2\cos(\underbrace{\pi x - \pi/4}) = 0$

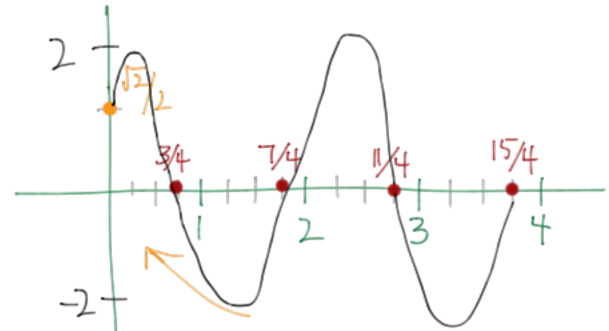
$= \pi/2,\ 3\pi/2,\ 5\pi/2,\ 7\pi/2,\ \ldots$

$\pi x - \pi/4 = \pi/2 \Rightarrow x = \frac{3}{4}$

$\pi x - \pi/4 = 3\pi/2 \Rightarrow x = 7/4$

$\vdots$

$x = 11/4,\ 15/4$



Now just plug in $x=0$ to see where to start: $y = 2\cos(0 - \pi/4) = \sqrt{2}/2$

amplitude $= 2$

5. Find the domain of the function.

$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 5x - 3}$$

Factor $2x^2 - 5x - 3$

ac: -6; -6, 1; b: -5

$2x^2 - 6x + x - 3$

$2x(x-3) + 1(x-3)$

$(2x+1)(x-3)$

Undefined when $(2x+1)(x-3) = 0$

$2x+1=0 \qquad x-3=0$

$x = -1/2 \qquad x = 3$

$(-\infty, -\frac{1}{2}) \cup (-\frac{1}{2}, 3) \cup (3, \infty)$

6. Find the exact values of all solutions x in the interval $[0, 2\pi)$ to the equation.

$$\sec^2 x - 6\tan x + 4 = 0$$

$(\tan^2 x + 1) - 6\tan x + 4 = 0$

$\tan^2 x - 6\tan x + 5 = 0$

$y = \tan x$

$y^2 - 6y + 5 = 0$

$(y-5)(y-1) = 0$

$y = 5, 1$

$\tan x = 1, 5$

$x = \pi/4, 5\pi/4$

can't do w/o calculator. My bad

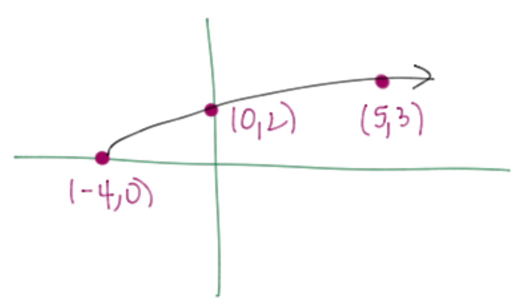
7. Find the domain and range of the function, and sketch it. Indicate the x and y-intercepts on the graph.

$$f(x) = \sqrt{x+4}$$

domain: $x + 4 \geq 0$

$x \geq -4$

range: $y \geq 0$



ANSWERS.

1. $\frac{\ln 17-\ln 15}{2\ln 15-\ln 17}$
2. $[0, \frac{\pi}{6}] \cup [\frac{5\pi}{6}, 2\pi]$
3. $(-\infty, -7) \cup (-3, \infty)$
4. $n - \frac{1}{4}$ for integers n.
5. $\cancel{(\infty, -\frac{1}{2}) \cup (\frac{1}{2}, 3) \cup (3, \infty)}$
6. $\frac{\pi}{4}$, $\frac{5\pi}{4}$
7. domain: $[-4, \infty)$, range: $[0, \infty)$, intercepts: $(0, 2)$, $(-4, 0)$