Precalculus Review 2, 09/08/14

1. Find a formula for the function $f(x)$ whose graph consists of all (x, y) such that
$$x = \frac{1+y}{1-y}.$$
What is the domain of f?

Answer:
$$\begin{aligned} x(1-y) &= 1+y \\ x - xy &= 1+y \\ -y - xy &= 1-x \\ y(-1-x) &= 1-x \\ y &= \frac{x-1}{x+1} \end{aligned}$$
The domain is $(-\infty, -1) \cup (-1, \infty)$.

2. Suppose $g(x)$ is defined such that $g(x+1) = x^3$. What is $g(0)$?

Answer: To find $g(0)$, we need to find the x value making $x+1=0$. This happens when $x=-1$. So $g(-1+1) = g(0) = (-1)^3 = -1$.

3. Let $f(x) = x - 4$ and let
$$g(x) = \begin{cases} \frac{x^2-16}{x+4} & \text{if } x \neq -4 \\ k & \text{if } x = -4 \end{cases}$$
Determine k such that $f(x) = g(x)$ for all x.

Answer: Firstly $\frac{x^2-16}{x+4} = x - 4$ by factoring the numerator. When $x = -4$, $x - 4 = -8$. So $k = -8$.

4. Let $f(x) = \sin^2(x)$. Why does $f(x)$ not have an inverse? What is an interval on which $f(x)$ has an inverse? What is the inverse?

   Answer: The graph of $f(x)$ is a bunch of "humps." It's easy to see this fails the horizontal line test. An interval on which it is invertible is $[0, \pi/2]$. To find the inverse, switch x and y and solve for y.

$$\begin{aligned} x &= \sin^2(y) \\ \sqrt{x} &= \sin(y) \\ \arcsin(\sqrt{x}) &= y \end{aligned}$$

5. Describe the set of points whose distance to $(6, 2)$ is the same as the distance to $(0, 0)$. What shape does it have?
   Answer: Use the distance formula:

$$\begin{aligned} \sqrt{x^2 + y^2} &= \sqrt{(x-2)^2 + (y-6)^2} \\ x^2 + y^2 &= (x-6)^2 + (y-2)^2 \\ x^2 + y^2 &= x^2 - 12x + 36 + y^2 - 4y + 4 \\ y &= -3x + 10 \end{aligned}$$

   The space if equidistant points is a line.

6. One of the following statements is right, and one is wrong. Which is which?

$$\arcsin(\sin q) = q$$

$$\sin(\arcsin q) = q$$

   Answer: Recall that the range of $\arcsin(q)$ is $[-\pi/2, \pi/2]$. Hence the first formula is only true if we restrict the domain of $\sin(q)$ to $[-\pi/2, \pi/2]$. The second formula is always true - $\arcsin(q)$ returns some angle whose sin is indeed the value q we started with.