Completing the Square

1. What number do you need to add to $x^2 + 10x$ to make it a perfect square? How about $x^2 + 14x$? How about $x^2 + 5x$? **Answer:**

$$x^2 + 10x + 25 = (x + 5)^2$$

$$x^2 + 14x + 49 = (x + 7)^2$$

$$x^2 + 5x + \left(\frac{5}{2}\right)^2 = \left(x + \frac{5}{2}\right)^2$$

2. Repeat the following problem for $x^2 + bx$ and $ax^2 + bx$. In both cases, write them in the form $C(x + h)^2 + k$. **Answer:**

$$x^2 + bx = \left(x + \frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2$$

$$ax^2 + bx = a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2$$

3. Now add/subtract appropriate amounts to express $ax^2 + bx + c$ in the form $a(x + h)^2 + k$. **Answer:**

$$ax^2 + bx + c = a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$$

4. Use the previous to derive the quadratic formula and find the formula for the vertex of a parabola. **Answer:**

$$\begin{aligned}
a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c &= 0 \\
a\left(x + \frac{b}{2a}\right)^2 &= -a\left(\frac{b}{2a}\right)^2 - c \\
a\left(x + \frac{b}{2a}\right)^2 &= \frac{ab^2 - 4a^2c}{4a^2} \\
\left(x + \frac{b}{2a}\right)^2 &= \frac{b^2 - 4ac}{4a^2} \\
x + \frac{b}{2a} &= \frac{\pm\sqrt{b^2 - 4ac}}{2a} \\
x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\end{aligned}$$