

Limits, Math 221 Do as many as you can!

1. Evaluate the limit $\lim_{x \rightarrow 4} x^4 - 4x + 1$.

2. Evaluate the limit $\lim_{x \rightarrow 3} \frac{3x^2 + 1}{\sqrt{x^3 + 9}}$.

3. Evaluate the limit $\lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^2 - 3x + 2}$.

4. Evaluate the limit $\lim_{x \rightarrow 4} \frac{x - 4}{\sqrt{x} - 2}$.

5. Evaluate the limit $\lim_{x \rightarrow 1} \frac{2x - 1}{8x^3 - 1}$.

6. Evaluate the limit $\lim_{x \rightarrow 2} \frac{\frac{1}{x} - \frac{1}{2}}{x - 2}$.

7. Evaluate the limit $\lim_{x \rightarrow 3} \frac{\sqrt{x} - 3}{x - 9}$.

8. Evaluate the limit $\lim_{x \rightarrow 3} \frac{\sqrt{x + 6} - 3}{x^2 - 9}$ or show that it does not exist.

9. Evaluate the limit $\lim_{x \rightarrow \infty} \frac{x^3 + 2x + 1}{3x^3 - x + 4}$ or show that it does not exist.
10. Evaluate the limit $\lim_{x \rightarrow -\infty} \frac{x^{17} - 7x^{2013}}{3x^2 + x^{2014}}$ or show that it does not exist.
11. Evaluate the limit $\lim_{x \rightarrow \infty} \frac{\sqrt{x^2 + 3x}}{2x}$ or show that it does not exist.
12. Evaluate the limit $\lim_{x \rightarrow -\infty} \frac{x - 3}{\sqrt{x^2 - 9}}$ or show that it does not exist.
13. Evaluate the limit $\lim_{x \rightarrow \infty} \sqrt{x^2 + x} - x$
14. Here are your tasks. Some might be impossible. Find functions $f(x)$ and $g(x)$ so that
- (a) $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x) = 0$ but $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = 1$.
 - (b) $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x) = 0$ but $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = 2$.
 - (c) $\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} g(x) = 0$ but $\lim_{x \rightarrow 0} \frac{f(x)}{g(x)} = \infty$.
 - (d) $\lim_{x \rightarrow 0} f(x) = 0, \lim_{x \rightarrow 0} g(x) = +\infty$ but $\lim_{x \rightarrow 0} f(x)g(x) = 3$.
 - (e) $\lim_{x \rightarrow 0} f(x) = 0, \lim_{x \rightarrow 0} g(x) = +\infty$ but $\lim_{x \rightarrow 0} f(x)g(x) = -3$.
 - (f) $\lim_{x \rightarrow 0} f(x) = 0, \lim_{x \rightarrow 0} g(x) = +\infty$ but $\lim_{x \rightarrow 0} f(x)g(x) = 3$.
 - (g) $\lim_{x \rightarrow 0} f(x) = \infty, \lim_{x \rightarrow 0} g(x) = -\infty$ but $\lim_{x \rightarrow 0} f(x) + g(x) = 4$.