Trig Limits, Math 221 Do as many as you can!

1. Recall that $\lim_{x\to 0} \frac{\sin(x)}{x} = 1$. Use this limit along with the other "basic limits" to find the following:

   (a) $\lim_{x\to 0} \frac{1-\cos(x)}{x^2}$. [Hint: Multiply top and bottom by $1+\cos(x)$.]

   (b) $\lim_{x\to 0} \frac{1-\cos(x)}{x}$.

   (c) $\lim_{x\to 0} \frac{\tan(x)}{x}$.

2. Evaluate the limit $\lim_{x\to \frac{\pi}{2}} \tan(x) - \sec(x)$ or show that it does not exist.

3. Evaluate the limit $\lim_{x\to 2014\pi} \frac{\sin(x+2013\pi)}{\sin(x)}$ or show that it does not exist.

4. Evaluate the limit $\lim_{x\to 0} \frac{\sin(x)}{1-\cos(x)}$ or show that it does not exist.

5. Evaluate the limit $\lim_{x\to 0} \frac{1-\cos(3x)}{2x^2}$ or show that it does not exist.

6. Evaluate the limit $\lim_{x\to 0} \frac{3x}{\sin(2x)}$ or show that it does not exist.

7. Evaluate the limit $\lim_{x\to 0} \frac{\sin(\sin(x))}{x}$.

8. Evaluate the limit $\lim_{x\to 0} \frac{1-\cos(\pi x)}{x^2}$.

9. Evaluate the limit

10. Evaluate the limit

11. Evaluate the limit

12. Evaluate the limit $\lim_{x\to 0} \frac{\sin(x)\sin(2x)}{\sin(3x)\sin(4x)}$.

13. Evaluate the limit $\lim_{x\to 0} \frac{1-\cos(5x)}{\sin^2(3x)}$.

14. Use the Sandwich Theorem to evaluate the limit $\lim_{x\to 0} x \cdot \sin\left(\frac{1}{x}\right)$.

15. Use the definition of the derivative to find the derivative of $f(x) = 3x+2$.

16. Determine where each of the following functions is continuous, and justify your answers:

   (a) $g(x) = \begin{cases} (x^2-1)/(x+1) & \text{for } x \neq -1 \\ 2 & \text{for } x = -1 \end{cases}$.

(b) $h(x) = \begin{cases} (x^2-1)/(x+1) & \text{for } x \neq -1 \\ -2 & \text{for } x = -1 \end{cases}.$

(c) $j(x) = \begin{cases} x^2 - 2x & \text{for } |x| > 1 \\ 3x - 2 & \text{for } |x| \leq 1 \end{cases}.$

(d) $p(x) = \begin{cases} \sin\left(\dfrac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}.$

(e) $q(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}.$

(f) $k(x) = \lceil x \rceil$, the "ceiling function" (which returns the smallest integer that is $\geq x$).

17. Find a value for a such that the function

$$f(x) = \begin{cases} \frac{6x^2-54}{x-3} & \text{for } x \neq 3 \\ a & \text{for } x = 3 \end{cases}$$

is continuous.

18. Find all values of a so that

$$f(x) = \begin{cases} \sin(x+a) & \text{for } x < 0 \\ ax^2 & \text{for } x \geq 0 \end{cases}$$

is continuous.

19. Find all asymptotes (horizontal, vertical, slanted) of the function $f(x) = \dfrac{3x^2+x-1}{x+3}$.

20. Find all asymptotes (horizontal, vertical, slanted) of the function $f(x) = \dfrac{x^2-1}{x^2+1}$.

21. Use the definition of the derivative to find the derivative of $g(x) = x^2$.

22. Use the definition of the derivative to find the derivative of $h(x) = \dfrac{2}{x}$.

23. Use the definition of the derivative to find the derivative of $f(x) = \sqrt{x+1}$.