Basic Integrals, Math 221 Do as many as you can!

1. Decide whether each of the following integrals makes sense and, if they do, evaluate them:
   (a) $\int x$
   (b) $\int_1^2 x$
   (c) $\int \, dx$
   (d) $\int_1^2 \, dx$
   (e) $\int x \, dx$
   (f) $\int x \, dt$
   (g) $\int t \, dx$
   (h) $\int d \, dx$
   (i) $\int d \, dd$
   (j) $\int dx \, dx$
   (k) $\int_0^x dx \, dx$
   (l) $\int_1^d dx \, dx$
   (m) $\int_1^{\triangle} dx \, dx$
   (n) $\int Cx \, dx$
   (o) $\int_{\triangle}^{x} x \, dd$
   (p) $\int_0^x dx \, dd$
   (q) $\int_{\triangle}^{\square} \star \, d\star$
   (r) $\int_{\triangle}^{\square} \square \, d\square$
   (s) $\int e^x dx$
   (t) $\int e^x de$
   (u) $\int \frac{1}{\text{cabin}} \, d(\text{cabin})$

2. Evaluate the integral $\displaystyle\int \frac{1 + x + x^2 + x^3 + x^4}{x^2} \, dx$.

3. Evaluate the integral $\int_0^1 3e^x \; dx$.

4. Evaluate the integral $\int_1^2 (\cos x + e^x + \frac{1}{x} + x + 1) \; dx$.

5. Evaluate the integral $\int_0^1 \frac{1}{\sqrt{1-x^2}} \; dx$.

6. Evaluate the integral $\int x^{2013} + \sqrt[2013]{x} + \frac{1}{\sqrt[2013]{x}} \, dx$.

7. Evaluate the integral $\int_0^\pi (e^x + \sin x) \; dx$.

8. Evaluate the integral using the Fundamental Theorem of Calculus.

   (a) $\int_{-1}^5 (1+3x)dx$

(b) $\int_0^2 (2-x^2)dx$

(c) $\int_1^2 x^3 dx$

9. Fact: If $f(x)$ is a function defined on an interval I, and one antiderivative of $f(x)$ on I is $F(x)$, then any other antiderivative of $f(x)$ on I is of the form $F(x)+C$ for some C. (This follows from the Mean Value Theorem.)

   (a) Find all the antiderivatives of x^2.

   (b) Show that the "most general antiderivative" of $\frac{1}{x}$ is $f(x) = \begin{cases} \ln(x)+C & \text{for } x>0 \\ \ln(-x)+D & \text{for } x<0 \end{cases}$, for some possibly different constants C and D. (In particular, neither $\ln(x)+C$ nor $\ln|x|+C$ is "the most general antiderivative" of $\frac{1}{x}$.)

10. If $f(x)$ is a function, we denote "the set of antiderivatives of $f(x)$" with the notation $\int f(x)\,dx$.

    Examples: $\int x\,dx = \frac{1}{2}x^2 + C$ and $\int (e^x + \sqrt{x})\,dx = e^x + \frac{2}{3}x^{3/2} + C$.

    (a) Make a list of basic antiderivatives.

    (b) Find $\displaystyle\int \left(x^2 + \sin(x) + e^x - 4 - \frac{1}{\sqrt[4]{x}} + \frac{1}{x}\right) dx$.

    (c) Find $\displaystyle\int \left(\cos(2x) + e^{3x} + \frac{1}{\sqrt{1-x^2}} + \frac{4}{x^2+1} - \sec(x)\tan(x)\right) dx$.