Basic Integrals, Math 221 Do as many as you can!

1. Decide whether each of the following integrals makes sense and, if they do, evaluate them:
   (a) $\int x$
   (b) $\int_1^2 x$
   (c) $\int dx$
   (d) $\int_1^2 dx$
   (e) $\int x \, dx$
   (f) $\int x \, dt$
   (g) $\int t \, dx$
   (h) $\int d \, dx$
   (i) $\int d \, dd$
   (j) $\int dx \, dx$
   (k) $\int_0^x dx \, dx$
   (l) $\int_1^d dx \, dx$
   (m) $\int_1^\triangle dx \, dx$
   (n) $\int Cx \, dx$
   (o) $\int_0^x x \, dd$
   (p) $\int_0^x dx \, dd$
   (q) $\int_\triangle^\square \, d\star$
   (r) $\int_\triangle^\square d\square$
   (s) $\int e^x dx$
   (t) $\int e^x \, de$
   (u) $\int \frac{1}{\text{cabin}} \, d(\text{cabin})$

2. Evaluate the integral $\int \frac{1 + x + x^2 + x^3 + x^4}{x^2} \, dx$. 
3. Evaluate the integral \( \int_0^1 3e^x \, dx \).

4. Evaluate the integral \( \int_1^2 (\cos x + e^x + \frac{1}{x} + x + 1) \, dx \).

5. Evaluate the integral \( \int_0^1 \frac{1}{\sqrt{1-x^2}} \, dx \).

6. Evaluate the integral \( \int x^{2013} + \frac{2^0}{\sqrt{x}} + \frac{1}{2013/2} \, dx \).

7. Evaluate the integral \( \int_0^\pi (e^x + \sin x) \, dx \).

8. Evaluate the integral using the Fundamental Theorem of Calculus.
   (a) \( \int_{-1}^5 (1 + 3x) \, dx \)
(b) $\int_0^2 (2 - x^2) \, dx$

(c) $\int_1^3 x^3 \, dx$

9. Fact: If $f(x)$ is a function defined on an interval $I$, and one antiderivative of $f(x)$ on $I$ is $F(x)$, then any other antiderivative of $f(x)$ on $I$ is of the form $F(x) + C$ for some $C$. (This follows from the Mean Value Theorem.)

(a) Find all the antiderivatives of $x^2$.

(b) Show that the “most general antiderivative” of $\frac{1}{x}$ is $f(x) = \begin{cases} \ln(x) + C & \text{for } x > 0 \\ \ln(-x) + D & \text{for } x < 0 \end{cases}$ for some possibly different constants $C$ and $D$. (In particular, neither $\ln(x) + C$ nor $\ln |x| + C$ is “the most general antiderivative” of $\frac{1}{x}$.)

10. If $f(x)$ is a function, we denote “the set of antiderivatives of $f(x)$” with the notation $\int f(x) \, dx$.

**Examples:** $\int x \, dx = \frac{1}{2} x^2 + C$ and $\int (e^x + \sqrt{x}) \, dx = e^x + \frac{2}{3} x^{3/2} + C$.

(a) Make a list of basic antiderivatives.

(b) Find $\int \left( x^2 + \sin(x) + e^x - 4 - \frac{1}{\sqrt{x}} + \frac{1}{x} \right) \, dx$.

(c) Find $\int \left( \cos(2x) + e^{3x} + \frac{1}{\sqrt{1-x^2}} + \frac{4}{x^2 + 1} - \sec(x) \tan(x) \right) \, dx$. 