

Review for Midterm 1

1. Find the constant  $c$  that makes  $g$  continuous on  $(-\infty, \infty)$ .

$$g(x) = \begin{cases} x^2 - c^2, & x < 4 \\ cx + 20, & x \geq 4 \end{cases}$$

2. Find the average rate of change for the function  $f(x) = x^3 + 1$  over the interval  $[2, 3]$ .

3. Find  $\lim_{h \rightarrow 0} \frac{3}{\sqrt{3h+1}+1}$

4. Determine  $\lim_{x \rightarrow -3} \frac{2 - \sqrt{x^2 - 5}}{x + 3}$

5. Prove using the  $\epsilon, \delta$  definition of limit that  $\lim_{x \rightarrow 6} \left(\frac{x}{4} + 3\right) = \frac{9}{2}$ .

6. Calculate the derivative of  $f(x) = x^2$  at the point  $x = 1$  using the definition of derivative.

7. Find  $\lim_{x \rightarrow \infty} \frac{2x+5}{x^2-7x+3}$

8. If  $f(x)$  is a continuous function and if  $x^4 \leq f(x) \leq x^2$  for  $-1 \leq x \leq 1$ , are there any points  $a$  such that you can determine  $\lim_{x \rightarrow a} f(x)$ ?

9. Find the domain and range of  $g(x) = \frac{1}{x+1}$

10. Prove using the  $\epsilon, \delta$  definition of limit that  $\lim_{x \rightarrow 4} 9 - x = 5$

11. Find  $\lim_{x \rightarrow 0} \frac{\sin 2x}{\sin x}$

12. Find  $\lim_{\theta \rightarrow 0} \frac{\sin \sqrt{2}\theta}{\sqrt{2}\theta}$

13. Find the derivative of  $f(x) = 15 - x + 4x^2 - 4x^4$

14. Find  $\lim_{x \rightarrow \infty} \frac{2}{x} - 3$

15. Calculate the derivative of  $f(x) = 3x - 4$  using the definition of derivative.

16. True or false: If  $\lim_{x \rightarrow a} f(x)$  exists, and  $\lim_{x \rightarrow a^+} f(x) = c$ , then  $\lim_{x \rightarrow a} f(x) = c$ .

17. Find the derivative of  $g(x) = (x^3 - 7)(2x^2 + 3)$

18. Determine  $\lim_{x \rightarrow 9} \frac{\sqrt{x}-3}{x-9}$

19. For what value of  $b$  is the function  $g(x) = \begin{cases} x, & x < -2 \\ bx^2, & x \geq -2 \end{cases}$  continuous?
20. Let  $f$  be the function defined by the requirement that for any  $x$  one has  $y = f(x)$  if and only if  $y$  is the largest of all possible solutions of  $2y^2 = -2x - 5xy$ . Graph  $f$  and find a formula. What is the domain and range?
21. Find the derivative of  $g(x) = (8x^2 - 5x)(13x^2 + 4)$
22. True or false: If  $f(x)$  and  $g(x)$  are continuous for  $0 \leq x \leq 1$ , the  $\frac{f(x)}{g(x)}$  is continuous for  $0 \leq x \leq 1$ .

23. Find the derivative of  $f(x) = \frac{4x-5}{3x+2}$

24. Find  $\lim_{x \rightarrow \infty} \frac{2x+3}{5x+7}$

25. Prove using the  $\epsilon, \delta$  definition of limit that  $\lim_{x \rightarrow 9} \sqrt{x-5} = 2$ .

26. Find  $\lim_{x \rightarrow 3} \frac{1}{(x-3)^2}$

27. Find an examples of functions  $f(x)$  and  $g(x)$  and a number  $a$  where  $\lim_{x \rightarrow a} f(x)g(x)$  exists, but  $\lim_{x \rightarrow a} f(x)$  and  $\lim_{x \rightarrow a} g(x)$  do not exist.

28. Find the derivative of  $f(x) = \frac{1}{1+x+x^2+x^3}$

29. Find  $\lim_{x \rightarrow \infty} \frac{\sin 2x}{1+x^2}$

30. Evaluate  $\lim_{u \rightarrow \infty} \frac{(2u+1)^3}{(3u^2+1)^2}$ .

31. Find  $\lim_{x \rightarrow 0} \frac{\tan 2x}{x}$

32. If  $f$  and  $g$  are functions such that  $f(2) = 3$ ,  $f'(2) = -1$ ,  $g(2) = -5$ , and  $g'(2) = 2$ , find  $h'(2)$  for

(a)  $h(x) = 2f(x) - g(x)$

(b)  $h(x) = 5f(x) + 3g(x)$

(c)  $h(x) = g(x)g(x)$

(d)  $h(x) = \frac{1}{f(x)+g(x)}$

33. Express the following limits as in the definition of limit.

(a)  $\lim_{t \rightarrow c} v(t) = K$

(b)  $\lim_{t \rightarrow b} f(t) = M$



34. Calculate the derivative of  $f(x) = \sqrt{x}$  at  $x = 7$  using the definition of derivative.

35. Find the derivative of  $g(x) = \frac{1}{\sin x \tan x}$

36. Determine  $\lim_{x \rightarrow -2} \frac{-2x-4}{x^3+2x^2}$

37. True or false: If  $f(s) = f(t)$ , then  $s = t$ .

38. Prove using the  $\epsilon, \delta$  definition of limit that  $\lim_{x \rightarrow 1} \frac{1}{x} = 1$ .

39. A function  $f$  is given that satisfies  $f(3x + 1) = x^3$ . Find  $f(x)$ .

40. Find  $\lim_{x \rightarrow 0} \frac{x + x \cos x}{\sin x \cos x}$

41. Prove using the  $\epsilon, \delta$  definition of limit that  $\lim_{x \rightarrow 1} 2x - 4 = 2$ .

42. True or false:

(a) For all real numbers  $x$ , we have  $\sin(\arcsin x) = x$

(b) For all real numbers  $x$ , we have  $\arcsin(\sin x) = x$

43. Find  $\frac{dy}{dx}$  if  $y = \frac{3x-1}{x^2}$

44. Find the average rate of change for the function  $f(x) = \sqrt{4x+1}$  over the interval  $[0, 2]$ .

45. Determine  $\lim_{h \rightarrow 0} \frac{\sqrt{3h+1}-1}{h}$

46. Find the second derivative of  $y = \frac{3x+4}{x+1}$

47. Prove using the  $\epsilon, \delta$  definition of limit that  $\lim_{x \rightarrow \sqrt{3}} \frac{1}{x^2} = \frac{1}{3}$ .

48. Find the derivative of  $f(\theta) = \frac{\sin \theta}{\theta}$

49. Find  $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$  when  $f(x) = \sqrt{2x}$

50. True or false: If  $\lim_{x \rightarrow 6} f(x)g(x)$  exists, then the limit must be  $f(6)g(6)$ .

51. Find  $\lim_{x \rightarrow 0} \frac{\tan 3x}{\sin 8x}$

52. Find the derivative of  $f(t) = t - t^2 \cos t$

53. Find  $\lim_{x \rightarrow 2} \frac{1}{x-2}$

54. Evaluate  $\lim_{x \rightarrow 4^+} \frac{3}{x-4}$  and  $\lim_{x \rightarrow 4^-} \frac{3}{x-4}$

55. Determine  $\lim_{x \rightarrow 5} \frac{x-5}{x^2-25}$

56. For what value of  $a$  if  $f(x) = \begin{cases} x^2 - 1, & x < 3 \\ 2ax, & x \geq 3 \end{cases}$  continuous?

57. True or false: If  $f(x)$  and  $g(x)$  are continuous at 0, then  $f(g(x))$  is continuous at 0.

58. Find the derivative of  $f(x) = 4 \cos x$

59. Find  $\lim_{x \rightarrow \infty} \cos x$

60. Find  $\lim_{x \rightarrow 0} \frac{\sin 4x}{\sin 6x}$

## ANSWERS

- |                                          |                                                                                                                                      |                                                            |
|------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------|
| 1.                                       | 24. $\frac{2}{5}$                                                                                                                    | 41.                                                        |
| 2. 19                                    | 25.                                                                                                                                  | 42. True or false:                                         |
| 3. $\frac{3}{2}$                         | 26. $\infty$                                                                                                                         | (a) T                                                      |
| 4. $\frac{3}{2}$                         | 27.                                                                                                                                  | (b) F                                                      |
| 5.                                       | 28. $f'(x) = -\frac{1+2x+3x^2}{(1+x+x^2+x^3)^2}$                                                                                     | 43. $\frac{dy}{dx} = \frac{-3x+2}{x^3}$                    |
| 6. 2                                     | 29. 0                                                                                                                                | 44. 1                                                      |
| 7. 0                                     | 30. 0                                                                                                                                | 45. $\frac{3}{2}$                                          |
| 8. -1, 0, 1.                             | 31. 2                                                                                                                                | 46. $\frac{2}{(x+1)^3}$                                    |
| 9.                                       | 32. (a) -4                                                                                                                           | 47.                                                        |
| 10.                                      | (b) 1                                                                                                                                | 48. $\frac{\theta \cos \theta - \sin \theta}{\theta^2}$    |
| 11.                                      | (c) -20                                                                                                                              | 49. $\frac{1}{\sqrt{2x}}$                                  |
| 12. 1                                    | (d) $-\frac{1}{4}$                                                                                                                   | 50. F                                                      |
| 13. $f'(x) = -1 + 8x - 20x^3$            | 33. (a) For every $\epsilon > 0$ ,<br>there is a $\delta > 0$ such<br>that if $0 <  t-c  < \delta$ ,<br>then $ v(t) - K  < \epsilon$ | 51. $\frac{3}{8}$                                          |
| 14. -3                                   | (b)                                                                                                                                  | 52. $f'(t) = t^2 \sin t - 2t \cos t + 1$                   |
| 15. 3                                    | 34. $\frac{1}{2\sqrt{7}}$                                                                                                            | 53. DNE (it approaches $\pm\infty$<br>from each direction) |
| 16. T                                    | 35. $-\csc x(1 + 2 \cot^2 x)$                                                                                                        | 54. $\infty, -\infty$                                      |
| 17. $g'(x) = 10x^4 + 9x^2 - 28x$         | 36. $-\frac{1}{2}$                                                                                                                   | 55. $\frac{1}{10}$                                         |
| 18. $\frac{1}{6}$                        | 37. F                                                                                                                                | 56.                                                        |
| 19.                                      | 38.                                                                                                                                  | 57. F                                                      |
| 20.                                      | 39.                                                                                                                                  | 58. $4 \sin x$                                             |
| 21. $g'(x) = 416x^3 - 195x^2 + 64x - 20$ | 40. 2                                                                                                                                | 59. DNE                                                    |
| 22. F                                    |                                                                                                                                      | 60.                                                        |
| 23. $f'(x) = \frac{23}{(3x+2)^2}$        |                                                                                                                                      |                                                            |