Short Note

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Systems arising from Vectors

The following problem arises in many situations:

Problem 1. *Find constants* $a_1, \cdots, a_n$ *such that*

$$a_1 v_1 + \cdots a_n v_n = w$$

for some vectors $v_1, \cdots, v_n$ *and* w.

In all such problems, you want to put the vectors $v_1, \cdots, v_n$ in a matrix as COLUMNS and then row reduce. Two examples are given below. Note, the book does it this way as well.

Example 1. *Determine if the following set of vectors is linearly independent.*

$$v_1 = (1, 3, -1, 0), v_2 = (0, 1, 2, -2), v_3 = (-2, 4, 1, 3)$$

To do this, we need to show that if $c_1 v_1 + c_2 v_2 + c_3 v_3 = 0$ then $c_1 = 0, c_2 = 0, c_3 = 0$. This is equivalent to

$$c_1(1, 3, -1, 0) + c_2(0, 1, 2, -2) + c_3(-2, 4, 1, 3) = (0, 0, 0, 0)$$

which is the system

$$\begin{aligned} c_1 + 0c_2 - 2c_3 &= 0 \\ 3c_1 + c_2 + 4c_3 &= 0 \\ -c_1 + 2c_2 + c_3 &= 0 \\ 0c_1 - 2c_2 + 3c_3 &= 0 \end{aligned}$$

which is the same as solving

$$\begin{bmatrix} 1 & 0 & -2 & | & 0 \\ 3 & 1 & 4 & | & 0 \\ -1 & 2 & 1 & | & 0 \\ 0 & -2 & 3 & | & 0 \end{bmatrix}$$

By row reducing, you will get the answer $c_1 = c_2 = c_3 = 0$. NOTE THAT THE VECTORS WENT IN AS COLUMNS. □

Example 2. *Determine if* v *is in the span of* w *and* x

$$v = (1, 0, 0), w = (1/3, -2, 1), x = (\pi, 0, 1)$$

Again, we need to solve $c_1 w + c_2 x = v$. This is equivalent to row reducing

$$\begin{bmatrix} 1/3 & \pi & | & 1 \\ -2 & 0 & | & 0 \\ 1 & 1 & | & 0 \end{bmatrix}$$

In the practice midterm solutions, I have row reduced this to the following:

$$\begin{bmatrix} 1 & 0 & | & 0 \\ 0 & 1 & | & 0 \\ 0 & 0 & | & 3 \end{bmatrix}$$

This implies that $a = 0, b = 0$... and then an INCONSISTENT row. This means that the w is not in the span. IF we had got a single solution, or infinitely many solutions, it would be in the span.

Elementary Matrices aka Problem 10

This is related to question 10 on your practice midterm. We did not spend a ton of time on this in discussion, but it was heavily emphasized in lecture.

I am not sure that the way I do this is exactly, how Owen does it, but here goes. All problems involving this look like the following:

Problem 2. *Given matrices, A, B find E such that $EA = B$.*

In such a problem, B will be some row operations on A. To find E use the following. Set E to be the identity, and then do the row operations you do to A to the identity.

Example 3. *Find E such that $EA = B$ where*

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & -3 \\ 1 & 0 & 1 \end{bmatrix}$$

Step 1) Note that B is obtained from A by flipping R_1 and R_2.

Step 2) So doing the same thing to the identity, $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

we get $E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Example 4. *Find E such that $EA = B$ where*

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 0 & -1 & 2 \\ 1 & 0 & 1 \end{bmatrix} B = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 2 & -3 \\ 1 & 4 & -7 \end{bmatrix}$$

Step 1) Note that B is obtained from A: by flipping R_1 and R_2, then computing $2R_2 - R_3$ on the new matrix

Step 2) So doing the same thing to the identity, we first get $\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

and then $2R_2 - R_3$ gives $E = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 2 & 0 & -1 \end{bmatrix}$

COMMENTS:

- MAKE SURE TO THIS ONE STEP AT A TIME.
- TO CHECK YOUR ANSWER, MULTIPLY EA AND MAKE SURE YOU GET B
- DO NOT TRY TO COMPUTE A^{-1}