

# Sage Homework

Lalit Jain

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This is a running document that will be updated weekly.

## Basic Sage

### Problem 1

Use Sage to rederive the quadratic formula. I.e. declare new variables  $a, b, c, x$  and use Sage to solve  $ax^2 + bx + c = 0$ .

## Rings

### Problem 2

Compute the quotient ring  $\mathbb{Q}[x]/(x^3 + 2x + 1)$ . Reduce  $x^4, x^5 + x^4, x^5 + 2x$ .

### Problem 3

1. Find an irreducible polynomial of degree two over  $\mathbb{Z}/2\mathbb{Z}$ .
2. Use this to generate the finite field of order 4.
3. Write out the multiplication table of this field. Try to do this in a slick way.
4. What is a generator of the multiplicative group?

### Problem 4

Is  $\mathbb{Z}[\sqrt{5}]$  a UFD? Use Sage!!!

## Programming

### Problem 5

Compute  $1 + 2 + \dots + 100$  using a `for` loop on the command line. Now do it by creating a function.

## Problem 6

In a separate file titled `matrixproduct.sage`, write a function that takes as input two matrices in the format `[[a,b],[c,d]]` and multiplies them.

As a comment sage has functionality to multiply matrices automatically. How do you do it?

For a bonus do this for arbitrary size matrices.

## Problem 7

Find the number of points of  $y^2 = x^3 - x$  over  $\mathbb{F}_{7^k}$  for  $k = 1, 2, 3$ .

## Problem 8

There exists exactly one Pythagorean triplet for which  $a + b + c = 1000$ . Find the product  $abc$ .

## Problem 9

Use list comprehensions to list all multiples of 3 or 5 but not both under 100.