Maltsky

MATH 114 – FINAL EXAM
May 13, 2013

Your name:

Circle your TA's name: Rui Wang Sid Kiblawi

- **Be sure to show your work and explain what you did. You will receive reduced or zero credit for unsubstantiated answers.**

- **No books or calculators. You may refer to notes you have brought on one sheet of paper, as announced in class.**

- **Circle your answers.**

problem	possible score	score
1	5	
2	5, 10	
3	5, 10, 5	
4	5, 5	
5	10	
6	10, 10	
7	10, 10	
8	10, 10	
9	10	
10	5, 5	
Total	140	

1. Find the domain of the function

$$f(x) = \frac{2x^2 + 5x - 3}{2x^2 - 5x - 3}$$

Write your answer as an interval or a union of intervals.

$2x^2 - 5x - 3 \neq 0$

$2x^2 - 5x - 3 = 0 \Rightarrow X = \frac{5 \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2 \cdot 2}$

$X = \frac{5 \pm \sqrt{25 + 24}}{4}$

$X = \frac{5 \pm \sqrt{49}}{4}$

$X = \frac{5 \pm 7}{4}$

$X = \frac{12}{4}, \frac{-2}{4}$

$X = 3, -\frac{1}{2}$

$\boxed{D: (-\infty, -\tfrac{1}{2}) \cup (-\tfrac{1}{2}, 3) \cup (3, \infty)}$

2. For the function $f(x) = \sqrt{x+4}$

a) Determine its domain and range.

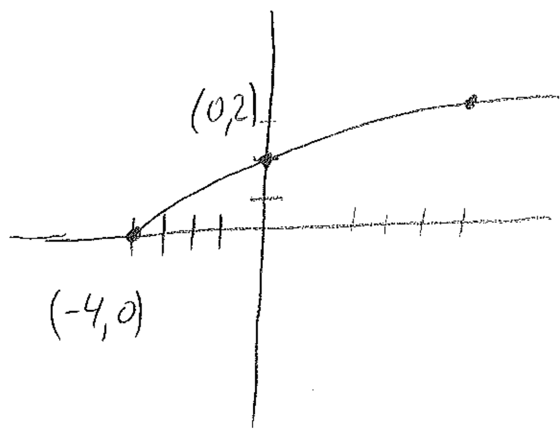
$\sqrt{x+4} \geq 0 \qquad x \geq -4$

$f^{-1}(x) = x^2 - 4 \qquad R: [-4, \infty)$

$D: [0, \infty)$

$\boxed{\begin{array}{l} D: [-4, \infty) \\ R: [0, \infty) \end{array}}$

b) Sketch the graph of the function. Indicate the x- and y- intercepts on the graph.



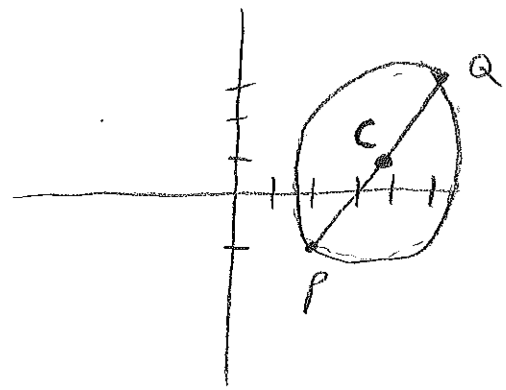
3. A circle has the points P(2, -1) and Q(5, 3) as the endpoints of a diameter.

a) Find the coordinates of the center C of the circle. Graph the circle, and indicate points P, Q, and C on the graph.

$$C = \left(\frac{P_x + Q_x}{2}, \frac{P_y + Q_y}{2}\right)$$

$$= \left(\frac{2+5}{2}, \frac{-1+3}{2}\right)$$

$$\boxed{C = (3.5, 1)}$$



b) Find the radius, area, and circumference of the circle.

$$d = |\overline{PQ}| = \sqrt{(5-2)^2 + (3+1)^2} = \sqrt{9+16} = \sqrt{25} = 5$$

$$r = \frac{d}{2} = \frac{5}{2}$$

$$A = \pi r^2 = \frac{25\pi}{4}$$

$$cir = \pi d = 5\pi$$

$$\boxed{\begin{aligned} r &= \frac{5}{2} \\ A &= \frac{25\pi}{4} \\ cir &= 5\pi \end{aligned}}$$

c) Find an equation of the circle.

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\boxed{(x-3.5)^2 + (y-1)^2 = \frac{25}{4}}$$

4. a) Solve the system of linear equations below

$$\begin{cases} 4x + y = 2 \\ 2x - 3y = 4.5 \end{cases}$$

$$4x + y = 2$$
$$-4x + 6y = -9$$
$$7y = -7$$
$$y = -1 \Rightarrow 4x - 1 = 2$$
$$4x = 3$$
$$x = \frac{3}{4}$$

$$\boxed{x = \frac{3}{4} \quad y = -1}$$

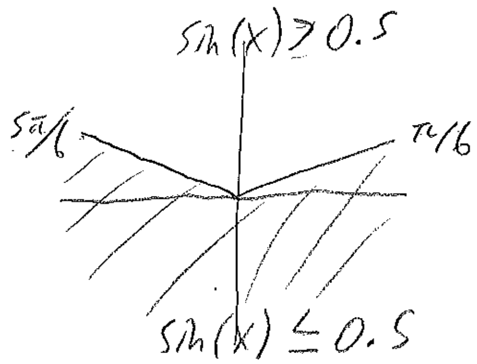
b) Find a number b such that the system of linear equations below

$$\begin{cases} 4x + y = 2 \\ -8x - 2y = b \end{cases}$$

has infinitely many solutions

$$8x + 2y = 4$$
$$-8x - 2y = b$$
$$0 = b + 4$$
$$\boxed{b = -4}$$

5. Find all values of x in the interval [0, 2π] that satisfy the inequality Sin x ≤ 0.5.
Write your answer as an interval or a union of intervals.



$$X = \left(0, \frac{\pi}{6}\right] \cup \left[\frac{5\pi}{6}, 2\pi\right]$$

6. Solve each equation for x.

a) $\ln(\ln x) = 1$

$$\ln(e) = 1 \Rightarrow \ln(x) = e$$

$$\boxed{x = e^e}$$

b) $\frac{1}{e^{7-4x}} = 6$

$$1 = 6 \cdot e^{7-4x}$$

$$e^{7-4x} = \frac{1}{6}$$

$$\ln e^{7-4x} = \ln \frac{1}{6}$$

$$7 - 4x = \ln \frac{1}{6}$$

$$-4x = \ln \frac{1}{6} - 7$$

$$\boxed{\begin{array}{l} x = \frac{7}{4} - \ln\left(\frac{1}{6}\right)^{1/4} \\ x = \frac{7}{4} + \ln\left(6^{1/4}\right) \end{array}}$$

$$\boxed{x = \frac{7 + \ln 6}{4}}$$

Any of these

7. Suppose a colony of bacteria starts with 100 cells and triples in size every two hours.

a) Find a function that models the population growth of this colony of bacteria.

$P = P_0 e^{r \cdot t}$

$P_0 = 100$

$\frac{P(2)}{P_0} = e^{2 \cdot r} = 3$

$r = \frac{\ln 3}{2}$

$P = P_0 e^{\frac{\ln 3}{2} \cdot t}$

$P = P_0 e^{\ln 3^{t/2}}$

$P = P_0 3^{t/2}$

$\boxed{P = 100 \cdot 3^{t/2}}$

b) How many cells will be in the colony after four hours?

$P(4) = 100 \cdot 3^{4/2}$

$P(4) = 100 \cdot 3^2$

$P(4) = 100 \cdot 9$

$\boxed{P(4) = 900}$

8. Simplify the expressions below as much as possible

a) $\sin x \cdot \{\frac{1}{1-\cos x} - \frac{1}{1+\cos x}\}$

$$\sin(x) \cdot \left(\frac{1+\cos x - (1-\cos x)}{1-\cos^2 x} \right)$$

$$\cancel{\sin(x)} \cdot \left(\frac{2 \cdot \cos x}{\sin^2 x} \right)$$

$$2 \cdot \frac{\cos(x)}{\sin(x)}$$

$$\boxed{2 \cdot \cot(x)}$$

b) $\sec^2 x - \tan^2 x$

$$\frac{1}{\cos^2(x)} - \frac{\sin^2 x}{\cos^2 x}$$

$$\frac{1-\sin^2(x)}{\cos^2 x}$$

$$\frac{\cos^2 x}{\cos^2 x}$$

$$\boxed{1}$$

9. Express cos(3x) as a function of cos x.

$$\cos(3x) = \cos(x+2x)$$

$$\cos(x+2x) = \cos(x)\cos(2x) - \sin(x)\sin(2x)$$

$$= \cos(x)\left(2\cos^2 x - 1\right) - \left(\sqrt{1-\cos^2 x}\right)\left(2\sin(x)\cos(x)\right)$$

$$= 2\cos^3 x - \cos x - \left(\sqrt{1-\cos^2 x}\right)\left(2\left(\sqrt{1-\cos^2 x}\right)\cos x\right)$$

$$= 2\cos^3 x - \cos x - 2\cos x\left(1-\cos^2 x\right)$$

$$= 2\cos^3 x - \cos x - 2\cos x + 2\cos^3 x$$

$$\boxed{\cos(3x) = 4\cos^3 x - 3\cos x}$$

10. For the polar equation $r \cos \theta = 4$

a) Write the equation in rectangular coordinates.

$r\cos\theta = x$

$\boxed{x = 4}$

b) Sketch the graph of the equation.